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Note on the Impedance of a Wire Grid Parallel to a Homogeneous Interface

JEFFREY L. YOUNG AND JAMES R. WAIT, FELLOW, IEEE

Abstract —**We provide new numerical data for the correction factor which is used to calculate the impedance of a planar wire grid parallel to the interface between two dielectric half-spaces. Comparisons are made with earlier investigations which clarify, extend, and supersede previous computations. Here we show more clearly the significant influence of the interface on the equivalent grid impedance.**

I. INTRODUCTION

A wire grid over a half-space is analogous to a transmission line with a shunt impedance. This shunt impedance consists of a logarithmic term modified by a correction factor, Δ. Depending on the value of h, the height of the grid above the interface, and d, the interwire spacing, this term may have a significant contribution to the total shunt impedance.

Previous results [1], [2] give only limited numerical data for Δ. We have found that these results have a few computational and drafting errors. We have reviewed the results in [1] and [2] and present here some graphs that illustrate important features not demonstrated in the earlier papers.

II. FORMULATION

With respect to a Cartesian coordinate system, the wire grid is contained in the plane $x = h$ and is parallel to a plane interface at $x = 0$. The grid is composed of an array of infinite wires parallel to the z axis and spaced a distance d between centers. The wires are taken to be of circular cross section and the diameter, $2a$, is assumed to be small compared to d. The media in both half-spaces are homogeneous and lossless with permittivity ϵ_1 for $x > 0$ and permittivity ϵ_2 for $x < 0$. The magnetic permeability is assumed to be the free-space value, μ_0, everywhere. A plane wave whose electric field is parallel to the z axis impinges on the grid with an incident angle of θ_0 as indicated in Fig. 1. Under the constraints of the thin wire approximation, the currents on each wire are assumed to be axially directed; from this we require that $\omega\sqrt{\mu_0\epsilon_1}\, a \ll 1$.

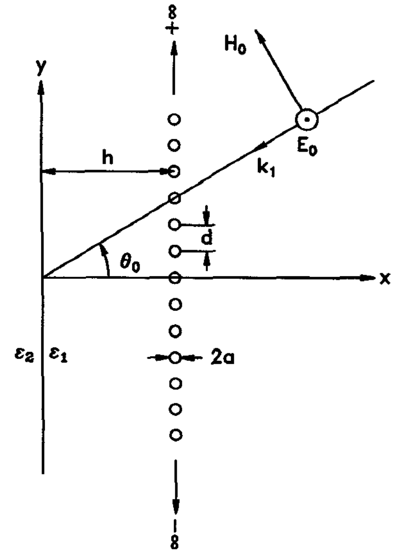

Fig. 1 The wire grid parallel to a half-space.

From previous analysis [2] it shown for this case that the equivalent shunt impedance, Z_g, is given by

$$Z_g = \frac{i\omega\mu_0 d}{2\pi}\left(\ln\frac{d}{2\pi a} + \Delta\right) + Z_w d. \quad (1)$$

Here Z_w is the axial impedance of the wire and Δ is the correction factor. The axial impedance can be expressed in terms of the modified Bessel functions as follows [3]:

$$Z_w = \frac{\eta_w}{2\pi a}\frac{I_0(\gamma_w a)}{I_1(\gamma_w a)} \quad (2)$$

where

$$\gamma_w = \sqrt{i\omega\mu_w(\sigma_w + i\omega\epsilon_w)} \quad (3)$$

and

$$\eta_w = \sqrt{\frac{i\omega\mu_w}{(\sigma_w + i\epsilon_w\omega)}}. \quad (4)$$

Also from [2], the correction term, Δ, is shown to be

$$\Delta = \frac{1}{2}\sum_{m=1}^{\infty}\left[\frac{1 + R_m \exp\left[-4\pi|h|d^{-1}\sqrt{(m + D_1\sin\theta_0)^2 - D_1^2}\right]}{\sqrt{(m + D_1\sin\theta_0)^2 - D_1^2}} + \frac{1 + R_{-m}\exp\left[-4\pi|h|d^{-1}\sqrt{(m - D_1\sin\theta_0)^2 - D_1^2}\right]}{\sqrt{(m - D_1\sin\theta_0)^2 - D_1^2}} - \frac{2}{m}\right] \quad (5)$$

where

$$R_m = \frac{\sqrt{(m + D_1\sin\theta_0)^2 - D_1^2} - \sqrt{(m + D_1\sin\theta_0)^2 - D_2^2}}{\sqrt{(m + D_1\sin\theta_0)^2 - D_1^2} + \sqrt{(m + D_1\sin\theta_0)^2 - D_2^2}} \quad (6)$$

and R_{-m} is obtained by replacing m with $-m$. We also have made use of the normalized dimensions D_1 and D_2, where $D_1 = d/\lambda_1$ and $D_2 = d/\lambda_2$, λ_1 and λ_2 being the wavelengths in the respective half-spaces. Here we should note that $D_1 =$

Manuscript received October 10, 1988; revised February 9, 1989
J. L. Young is with the Electronic/Microwave Department, Hughes Aircraft Company, Tucson, AZ 85734
J. R. Wait is with the Electromagnetics Laboratory, Department of Electrical and Computer Engineering, University of Arizona, Tucson, AZ 85721.
IEEE Log Number 8928199.

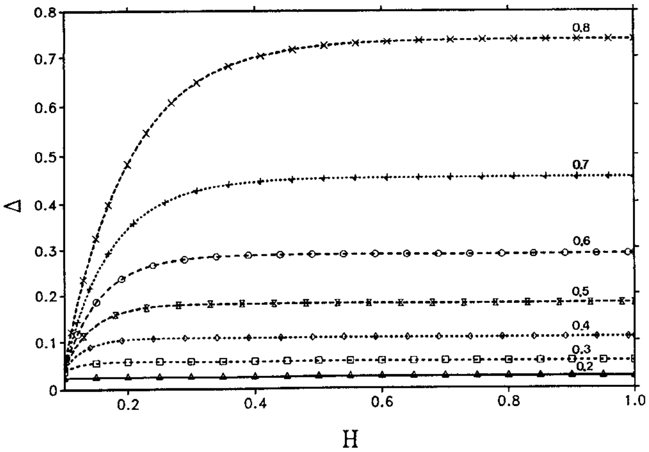

Fig. 2. The correction factor, Δ, as a function of the distance of the grid to the perfectly conducting plane for $\theta_0 = 0$ and different values of D_1.

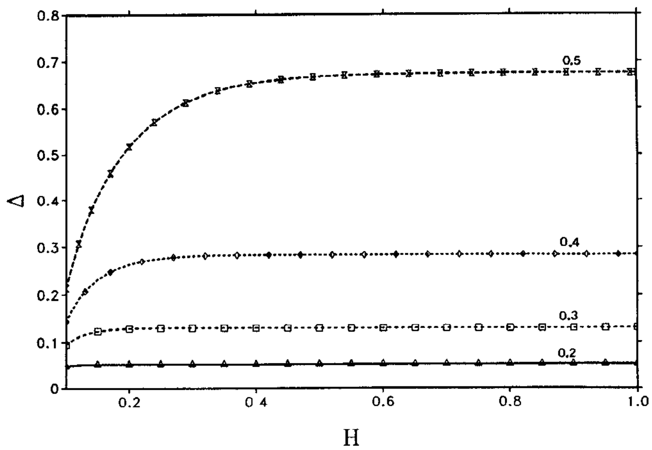

Fig. 3. The correction factor, Δ, as a function of the distance of the grid to the perfectly conducting plane for $\theta_0 = 45°$ and different values of D_1.

$\omega\sqrt{\epsilon_1\mu_0}\,d/2\pi$ and $D_2 = \omega\sqrt{\epsilon_2\mu_0}\,d/2\pi$. The shunt impedance of the grid when placed below the interface is obtained by interchanging the subscripts 1 and 2 in (5) and (6). For later convenience, we define the relative refractive index, N, by the relation $N = \sqrt{\epsilon_2/\epsilon_1} = \lambda_1/\lambda_2$ and the normalized height, H, by the relation $H = h/\lambda_1$ for both positive and negative h.

Before investigating the numerical characteristics of Δ, we will look at two limiting cases. We will denote by Δ^{∞} and $\Delta^{-\infty}$ the value of the correction factor when $h \to \pm\infty$, respectively. When $\theta_0 = 0$, we find from (5) that

$$\Delta^{\infty} = \sum_{m=1}^{\infty} \frac{1}{\sqrt{m^2 - D_1^2}} - \frac{1}{m} \tag{7}$$

and

$$\Delta^{-\infty} = \sum_{m=1}^{\infty} \frac{1}{\sqrt{m^2 - (ND_1)^2}} - \frac{1}{m}. \tag{8}$$

Letting $D_1^4 \ll 1$ and $(ND_1)^4 \ll 1$, we find that the difference of (7) from (8) is well approximated by

$$\Delta^{\infty} - \Delta^{-\infty} \approx 0.601 D_1^2 (1 - N^2) \tag{9}$$

which is consistent with a similar formula given in [2].

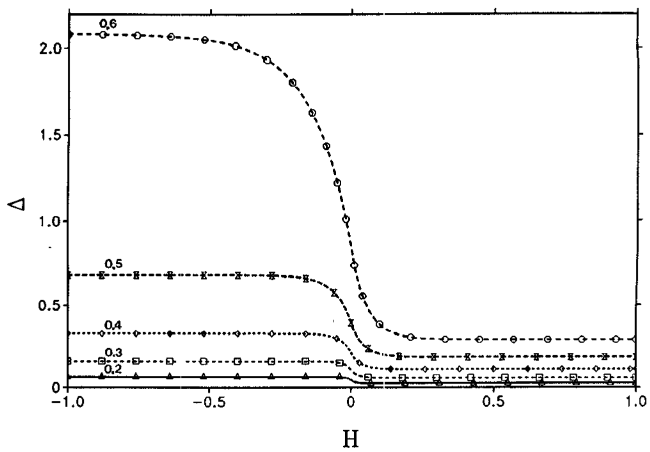

Fig. 4 The correction factor, Δ, as a function of the distance of the grid to the dielectric half-space for $\theta_0 = 0$, $N = 1.57$, and different values of D_1.

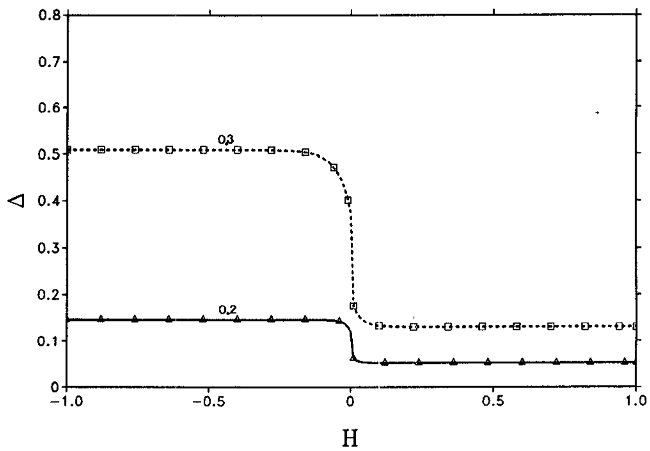

Fig. 5. The correction factor, Δ, as a function of the distance of the grid to the dielectric half-space for $\theta_0 = 45°$ $N = 1.57$, and different values of D_1.

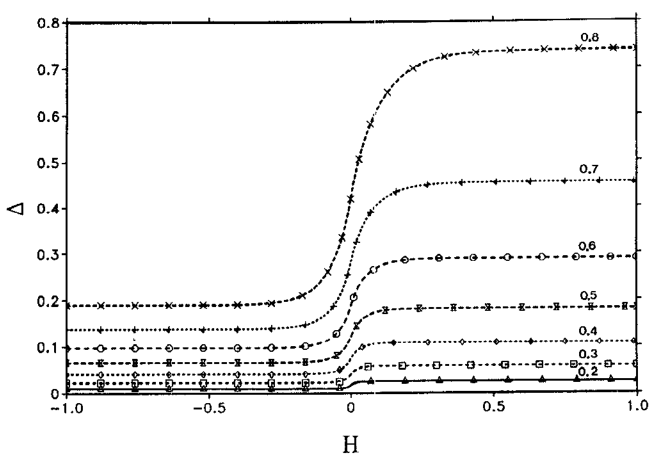

Fig 6. The correction factor, Δ, as a function of the distance of the grid to the dielectric half-space for $\theta_0 = 0$, $N = 1/1.57$, and different values of D_1

Although the above equations are completely general for all values of D_1, we will in subsequent computations restrict the value of D_1 such that the relations $D_1 < (1/(1 + \sin|\theta_0|)$ and $D_1 < (1/N(1 + \sin|\theta_0|)$ are satisfied. That is, we will not allow grating lobes to occur for all values of d and h.

III. Numerical Results

Letting the lower half-space be perfectly conducting, the correction fact, Δ, is plotted in Figs. 2 and 3 as a function of H for various values of D_1 when $\theta_0 = 0$ and 45°, respectively; the results are in general agreement with those sketched in an earlier study [1]. As expected, the values of Δ for $H \rightarrow \infty$ approach those results furnished by MacFarlane [4] where he assumed the grid was located in free space.

Similar results are shown in Figs. 4 and 5 when $N = 1.57$. Fig. 6 shows the correction factor when region 1 is the denser medium (i.e., $N = 1/1.57$) and $\theta_0 = 0$. The general shapes of the curves shown in Figs. 4, 5, and 6 are predicted by (9), and the values of Δ for $H \rightarrow \infty$ approach those of MacFarlane. The numerical data in Fig. 6 supersede and extend an earlier investigation [2].

IV. Concluding Remarks

We have extended and updated various results for the correction term used in computing the impedance of a wire grid parallel to a dielectric interface. We stress that the results herein are restricted to the case where the electric field is always parallel to the grid's wires.

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Analysis of Wide Transverse Inductive Metal Strips in a Rectangular Waveguide

BRIGHT H. CHU AND KAI CHANG, SENIOR MEMBER, IEEE

Abstract —**An analysis based on the variational method and the moment method has been developed to calculate the discontinuity susceptance due to one or more inductive strips in a rectangular waveguide. The strips can have wide widths and be located unsymmetrically on the transverse plane of the waveguide. The current distribution on the strips has been determined by solving a set of linear equations. The theoretical results agree closely with experiments.**

I. Introduction

Waveguide inductive strip discontinuities have many applications in waveguide filters and matching networks. Many analyses have been reported for single and multiple inductive strips in waveguide [1]–[7]. Marcuvitz provided closed-form formulas for the reactance of a transverse inductive strip with a broad range of widths [1]. The solution is only valid for a centered strip and only one strip can be considered. For a narrow strip, a solution based on the variational method can be found in Collin [2]. The strip location is not limited to the center of the waveguide. Chang and Khan [3], [4] reported an analysis for a two- and a three-strip discontinuity using the variational method. The current density ratios between the strips have also been determined. The analysis is limited to narrow strips since a constant current distribution is assumed on each strip. Furthermore, as the number of strips increases, the analysis becomes very complicated and a large number of nonlinear equations for current ratios are difficult to solve. Lewin reported a method of solving a general unsymmetrical multiple-strip geometry by using a singular-integral equation over a multiple interval [5], [6]. The method was based on the singular-integral equation approach which had previously been applied to a symmetrical double inductive aperture [7]. The analysis is quite general but no numerical results were reported.

The moment method has recently been used for the analysis of a probe-excited waveguide [8] and a single inductive metal post [9]. Analyses for inductive posts and diaphragms have also been reported [10], [11].

This paper reports an analysis based on variational and moment methods to calculate the discontinuity susceptance due to both a single strip and multiple strips in waveguide. The strips are located unsymmetrically on the transverse plane of a waveguide and the widths of the strips can be large. The current distribution on each strip is approximated by pulse expansion functions. The moment method is first used to determine the amplitudes of these expansion functions by solving an integral equation. Once the current distribution is determined, the variational method is used to calculate the discontinuity susceptance. Analyses for both single and multiple strips are given. Theoretical results have been compared with experiments for single wide centered strips, a single wide off-centered strip, a two-strip obstacle, and a three-strip discontinuity. The agreement is very good.

II. Analysis of a Single Wide Strip

It is assumed that the transverse metal strip is located at $z = 0$, as shown in Fig. 1. The strip is assumed to be infinitesimally thin and made of perfect conductor. By the requirement that the tangential E field vanish on the perfectly conducting strip surface S, we have [2]

$$\sin\frac{\pi x}{a} - \frac{j\omega\mu_0}{a}\sum_{n=1}^{\infty}\frac{1}{\Gamma_n}\sin\frac{n\pi x}{a}\int_S \sin\frac{n\pi x'}{a}J(x')\,dx' = 0 \quad (1)$$

where $J(x')$ is the current distribution on the strip, a is the width of the rectangular waveguide, and the integration is performed on the strip surface S. The propagation constant Γ_n is defined as

$$\Gamma_n = \frac{\pi}{a}\sqrt{n^2 - (2a/\lambda_0)^2}.$$

To solve the unknown current $J(x)$, it can be expressed in terms of pulse expansion functions as

$$J(x) = \sum_{q=1}^{N} I_q f(x_q) = \sum_{q=1}^{N} I_q\left[U(x - x_q) - U(x - x_{q+1})\right] \quad (2)$$

where N is the total number of segments for the pulse expansion function, $U(x)$ is the step function, and I_q is the amplitude. The index q indicates the qth segment along the strip. Equation (1)

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The authors are with the Department of Electrical Engineering, Texas A&M University, College Station, TX 77843-3128.

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